

Cognitive Computer Tools in the Teaching and Learning of Undergraduate Calculus

Nathan Borchelt, Ph.D.
Assistant Professor of Mathematics
Clayton State University

Department of Mathematics
2000 Clayton State Blvd.
Morrow, GA 30260
(678) 466-4424
NathanBorchelt@clayton.edu

The purpose of this study was to explore the use of a cognitive computer tool by undergraduate calculus students as they worked cooperatively on mathematical tasks. Specific attention was given to levels of cognitive demand in which the students were engaged as they completed in-class labs with the assistance of MathCAD. Participants were assigned to eight heterogeneous working groups consisting of four students each. One group was chosen as the focus of the case study. Data included student questionnaires, individual interviews, assignments, audio transcriptions of student discussions, and video recordings. Open and axial coding was used to analyze the data. Participants believed that the use of MathCAD allowed them to explore mathematics, spend more time on interpreting results, and focus on understanding. The cognitive computer tool reduced the reliance on low-level thinking skills and allowed for creativity in problem solving, permitting students to move toward high levels of thinking.

Keywords: Technology, Cognition, Mathematics, Cooperative Learning, Computer Tool

Introduction

The influx of technology into the college classroom has been inevitable, and the use of computer algebra systems in college level mathematics is becoming increasingly common. However, there is a minimal amount of research addressing the impact of a computer algebra system on developing mathematical understanding (Zbiek, 2003). In order for sound decisions to be made regarding pedagogy, research must be undertaken that explores how students use cognitive computer tools in the college classroom as part of a learning process. This study is important because the results can affect decisions made concerning the use of such software programs in undergraduate mathematics.

Although using technology in the classroom is not a new concept, implementing it as a cognitive tool definitely is (Jonassen & Reeves, 1996). Software must be used in ways related to mathematical thinking in order for it to be considered a cognitive computer tool. But, what exactly are the distinguishing characteristics of cognitive computer tools in mathematics education? Pea (1987) suggests that there are five general categories of process functions that are identified with cognitive technologies. Cognitive computer tools in mathematics should:

- 1) provide support for developing conceptual fluency,
- 2) aid in mathematical exploration,

- 3) allow for integration of different mathematical representations,
- 4) promote learning how to learn, and
- 5) encourage learning of problem-solving methods.

The primary purpose of this study (Borchelt, 2004) was to explore the use of a cognitive computer tool by undergraduate calculus students as they work cooperatively on mathematical tasks. The following primary question guided this study: In what ways does the use of a cognitive computer tool affect the level of cognitive demand on students in undergraduate calculus within the context of small group mathematical tasks? In order to understand this, it is important to consider two secondary questions: (1) What is the nature of students' use of the cognitive computer tools? (2) What are the students' perceptions about how the use of the cognitive computer tool contributes to their learning?

Teaching within a social constructivist framework calls for instructors to provide students with socially rich environments in which to explore mathematical content. Using a cognitive computer tool in the classroom as an integral part of teaching and learning may support such exploration. Jonassen and Reeves (1996) suggest that cognitive tools are most effective when they are applied within constructivist learning environments. There is a need for research to be conducted to examine whether or not the use of such technology supports high-level thinking in college mathematics classrooms. This study examined undergraduate calculus students as they used a cognitive computer tool to complete mathematical tasks while working in a cooperative group. A Mathematical Tasks Framework (Stein, Smith, Henningsen, & Silver, 2000) was used to study the levels of cognitive demand in which the students engage.

Theoretical Framework

Learning is determined, at least in part, by the interactions among a learner's existing knowledge, established social context, and the problem to be solved (Tam, 2000). It is through the interaction with objects and events that the learner constructs understanding. This is why the environment or context in which the learning takes place is so important. It provides the arena necessary to nurture the natural curiosity of the student. Noddings (1990) claims that constructivists generally agree that:

- 1) All knowledge is constructed.
- 2) There exist cognitive structures that are activated in the processes of construction.
- 3) Cognitive structures are under continual development.
- 4) Acknowledgement of constructivism as a cognitive position leads to the adoption of methodological constructivism.

Vygotsky (1978) argued that higher order thinking developed first in action and then in thought. He proposed that the potential for cognitive development is optimized within a "zone of proximal development" or an area of exploration for which a student is cognitively prepared, but requires assistance through social interaction. He emphasized the belief that learning is fundamentally a socially mediated activity and not a statically occurring phenomenon. The theory of social constructivism focuses on socially co-constructed knowledge formed through the interactions between members of a group or community and the world around it. Vygotskian social learning theory suggests an expert teacher be present who acts as an active participant in the social aspect of the classroom. From a constructivist perspective, the instructor's primary responsibility as facilitator is to establish

a collaborative environment where students are allowed to construct knowledge (Tam, 2000). The shared commitment and responsibility to inquiry between teacher and students play a vital role in the learning process.

Mathematical knowledge is created through reflection of abstract meaning (Noddings, 1990). This is to say that learning mathematics is an active, social process by which each student can develop his or her own understandings of concepts based on applications of previously obtained knowledge. "Social environments that establish an *interactive social context* for discussing, reflecting upon, and collaborating in the mathematical thinking necessary to solve a problem also motivate mathematical thinking" (Pea, 1987, p. 104). Students need opportunities on a regular basis to engage in learning experiences that enable the construction of deeper understanding regarding mathematical processes, concepts, and relationships. Activities in a mathematics classroom should engage students at high levels of mental reasoning. Stein, Smith, Henningsen, and Silver (2000) created a Mathematical Tasks Framework by which the different phases of a mathematical task can be analyzed. This conceptual framework classifies tasks based on the level of cognitive demand that is required of students throughout a task. The tasks can be evaluated at several stages beginning with the writing of a task through its implementation. Figure 1 provides a representation of how mathematical tasks unfold during classroom instruction (Stein & Smith, 1998).

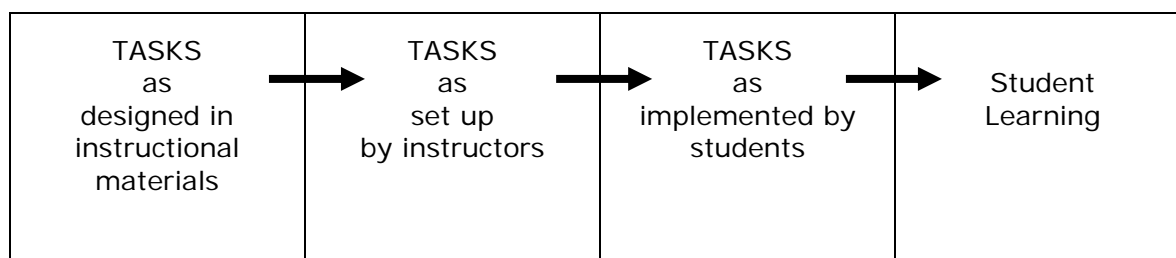


Figure 1. Mathematical Tasks Framework

The framework categorizes four levels of cognitive demand (Stein & Smith, 1998). The lowest levels are Memorization and Procedures Without Connections. The level of Memorization describes tasks that involve reproducing information such as facts or formulas. There is no connection to concepts or meanings that underlie the reproduced information. At the level of Procedures Without Connections, a task is very procedural and the approach to solving the problem is immediately evident. The focus at this level is on producing correct answers and the work does not require any explanation. The second two levels of cognitive demand are Procedures With Connections and Doing Mathematics. These categories require high levels of mental reasoning. At the level of Procedures With Connections, a task must focus student attention on the use of procedures for developing deep understanding of mathematical concepts, involve multiple representations, and require some degree of cognitive effort. At the level of Doing Mathematics, a task has the following characteristics: complex thinking, exploration of relationships, applications of previous knowledge, and some level of anxiety.

The Mathematical Tasks Framework was created to guide the analysis of mathematical tasks used in classrooms. This study analyzes student levels of cognitive demand as technology facilitated mathematical tasks are implemented within cooperative groups. In this study attention will be given to the role of the cognitive computer tool in effecting levels of

cognitive demand. The framework was to determine if the level of cognitive demand increases to a higher level, is maintained at the level at which it is written, or declines to a lower level.

Review of Literature

Anand and Zaimi (2000) compared the impact of using alternate methods of technology on achievement in the college classroom. Their study found that despite the insistence of students to use computers in the course, many of them demonstrated a good deal of resistance to using the technology. The instructor in this study had two goals to accomplish that were important to the integration of technology into a course. These were creating positive student attitudes towards the use of technology and empowering the student to take charge of learning. Developmental differences between adult learners and traditional students were quite apparent. The traditional college students involved in the study were not used to working independently. These students seemed to struggle with being responsible for communicating ideas via the technology used. Another important lesson learned by the researchers was that the use of technology required the students to work closely with an instructor. The use of technology encouraged more discussions between teacher and student concerning concepts and problems than what was usually the case for classes implementing traditional modes of instruction.

Rochowicz (1996) found that instructors who used technology as an integral part of the teaching and learning in their courses perceived an increase in more active learning. Interpretation of the results from the mathematical tasks became more necessary. A majority of the respondents also agreed that student motivation in the learning of calculus topics improves. This strongly suggests that the discourse generated from the use of technology greatly enhances the learning experience. "The perceived impact of using technology on specific topics of calculus appears to shift the focus of learning from symbol manipulation and skills to more interpretation, approximation, graphing, and modeling" (Rochowicz, 96, p.426). It was concluded that a technology enhanced calculus course did appear to be more conceptual, relevant, and meaningful for the students enrolled. Eighty-five percent of the teachers surveyed in the study either agreed or strongly agreed that computing technology use in calculus instruction requires significantly more time from the teacher and more creative teaching on the part of the educator. This is probably attributed to the fact that there is an increasing need for a more flexible, less structured pedagogy.

Pitcher (1998) conducted a study that addressed the effectiveness of computer-enhanced coursework in undergraduate mathematics. A group of educators in the United Kingdom worked to develop computer assisted learning modules for use in entry-level university mathematics courses. Students who participated in this study provided individual feedback by completing questionnaires and interviews that focused on a specific learning module. The computer-assisted learning module that was used was to assist students in learning the precalculus topic of complex numbers. In the analysis of the data for this study, a very important theme evolved. It is evident that the instructor must teach students how to harness the power of computer software and discover how it can work for them to reinforce or change their conceptual understanding of mathematics. Pitcher argues that in order for computer assisted learning to work effectively we must raise the expectations of our students and strengthen the learning experiences with adequate study skills training. Pitcher concluded that it is important to develop materials for advising the students on how to get the most out of the computer-assisted learning experience.

Ganguli (1992) studied college students in an intermediate level algebra course that were taught mathematical concepts with a computer as a teaching aid. It was determined that the students demonstrated an increased positive self-concept toward their mathematical abilities. Students in the computer enhanced class also demonstrated stronger motivation for doing mathematics than in similar courses where the technology was not incorporated into the learning process. Budenbender, Frischauf, Gogvadze, Melis, Libbrecht, and Ullrich (2002) looked at the experiences of university students while using a tutoring program called ActiveMath which integrates a computer algebra system. Their observations indicated an increase in student motivation to learn. The results were also positive in engaging students through exploratory learning.

The purpose of a study conducted by Foletta (1994) was to describe the nature of inquiry of four ninth and tenth grade high school students as it arose in high school geometry. Students used Geometer's Sketchpad to assist in their classroom investigations. She observed how the cognitive computer tool influenced student learning as within guided inquiry. Foletta defines guided inquiry as an instructional model that includes some form of discovery learning with discourse. The study sought to characterize the impact of the software tool on group interactions and discussions. The use of the software involved students in the process of modeling, conjecturing, verifying, and engaging in mathematical discourse. To investigate the small-group interactions and inquiry skills of the students, a case study approach was used. The case consisted of two female students and two male students along with the teacher. The sources of data came from transcripts of group observations, student interviews, student lab work, self-assessment surveys, student mathematics beliefs surveys, copies of student written work, and teacher interviews. The use of Geometer's Sketchpad seemed to have two effects on the students. First of all, the use of the software tool seemed to have a focusing effect for students. It encouraged students to spend more time-on-task. The software tool appeared to give confidence to the low achieving student within the context of guided inquiry activities because the small-group discussions or discourse resulted in the students working within their zone of proximal development. At the same time the researcher noticed somewhat of a scattering effect. Often the subjects had a hard time making connections from the technology output and the mathematical concepts that they were studying. If a scattering effect does exist, maybe this is something that could generate even further discussions about what the technology produces and why.

Confrey, Smith, Pintero, and Rizzuti (1991) studied peer interactions among twelfth grade students. The study focused on the interactions that were centered on use of a software tool called Function Probe. This software allows students to explore mathematical ideas by creating graphical, numerical, and symbolic representations. The tool was independent of the curriculum being used in the same way that the software used in the present study is independent of the curriculum used. Small group interactions were recorded and data were collected from both structured and unstructured student interviews. One major conclusion drawn from this research was that the computer has the potential to serve as a communicative tool within group activities. Students were able to develop a mutually understood framework of mathematical meaning based on explorations of concepts.

It is important to look at existing research regarding the use of computer technology in mathematics education. While there are studies involving the use of software by students, the research involving the impact of a CAS on student learning is minimal and fractured at best (Zbiek, 2003). The research that exists varies a lot in scope and most involve the use

of a handheld CAS such as the features available on the TI-92 calculator. Still, there have been some relevant studies that helped to inform the study described in this paper. The literature suggests an increase in student motivation and active learning in the classroom. The technology appears to serve as communicative tools for exploration, especially in the context of small-group tasks. It is also important to note that these studies suggest that the instructor plays a very important role in the technology enhanced environment.

Methods

Site and Participants

The research takes place at a four-year state university in the southeastern United States. The institution is located on the outskirts of a large metropolitan city and is primarily a commuter campus. During the first week of class, all students were provided with a brief explanation of the research and all were invited to complete a consent form indicating their willingness to volunteer as participants in the study. All participants were informed that they could opt out of the study at any time with no impact on their grade or status in the course. Early in the semester, students were placed into cooperative working groups of four using purposeful sampling procedures to assure a heterogeneous mix of students. Heterogeneous groups were used because they support more elaborative thinking, participation in mathematical discussions, and improve the quality of reasoning (Johnson, Johnson, and Holubec, 1993). Students with various levels of expertise and experiences were assigned together so that they could help each other learn and grow together in their mathematical understanding. The purposeful selection process ensured that each group was fairly representative of the types of students in the class. Within these cooperative learning groups, students completed in-class assignments or labs, which focused on the exploration or application of calculus concepts. An even number of members in the group was desirable to eliminate the isolation of one member from any discussion (Crabill, 1990). To obtain necessary background information on each student, an entrance questionnaire was administered to the participants during the first week of the academic semester. Academic performance in previous coursework, race, and gender were also considered. All groups were observed initially, but by the third week of classes, one group was chosen to serve as the specific case to be studied.

Teacher Researcher

An ethnographic study must emphasize direct personal involvement in the community; therefore the researcher acted as a participant observer in the classroom environment throughout the duration of this study. Some might argue that this presents a disadvantage due to the fact that the researcher is too involved with the participants, resulting in bias. On the contrary, while teacher research combines theory with practice, it still involves systematic inquiry through an emic perspective (Bauman & Duffy, 2001). An emic perspective provides a descriptive account of behavior in terms meaningful to the researcher; hence it is specific to the culture being studied. The teacher researcher has the ability to raise pertinent questions about what they believe and observe as part of the learning processes that occur in the classroom (MacLean & Mohr, 1999). This method empowers teachers to make a positive difference in classroom practices and provides relevant information about teaching and learning in actual classrooms (Ritchie, 2001, online). Teacher-researchers have a power to understand and critically examine the learning process.

There are some definite advantages in using teacher-research for this study. Since the data collection took place in a natural academic setting, access into the community being studied was not difficult. The instructor for a course is one of the most informed individuals in a classroom setting. Therefore, their participation provides an important backdrop for the study. Close analysis of student learning required that the researcher know the students through well-developed relationships. The student-teacher relationships built a trust that facilitated the ability to obtain meaningful data and feedback. This methodology provided valuable insights into how the use of the cognitive computer tool impacted the student levels of cognitive demand during a mathematical task.

Data Collection

The data collection took place for the entirety of one semester within a specific section of a first semester undergraduate calculus course. The use of MathCAD by the students as part of their learning experience and their interactions during mathematical tasks was the focus of the data collection. The data collected for this study came from several sources. Instrumentation for the study included: entrance and exit surveys, individual student interviews, video recordings of classroom activities, field notes based on classroom observations, and examples of student work. The information for answering the primary research question came from the group transcriptions, student interviews, and video recordings. The examples of student work and field notes helped to address the nature of students' use of the cognitive computer tools during the tasks. Students' beliefs about how the use of the cognitive computer tool contributes to their learning were documented using the surveys and interviews. These multiple sources of information together contribute to the process of understanding and describing the case that is being studied (Merriam, 1998).

Field observations took place during four calculus labs that were administered at various points throughout the semester. Each of these labs was designed to be completed by the groups during a regular seventy-minute class period. The labs were administered electronically using MathCAD. This software combines the power of a computer algebra system, graphing utility and mathematical word processor into a single electronic worksheet environment. Students were asked to use the software to solve challenging problems and explore concepts of calculus. Groups were also expected to show all of their work and give thorough justifications of their problem-solving processes using complete sentences. The completed student work helped document the accounts of progress in the classroom environment and convey their understanding of the concepts. All of the questions used for the labs were piloted during previous semesters to ensure their readability and appropriate level of difficulty. The labs provided the participants with applications of concepts already discussed in class and exploratory investigations into concepts not specifically presented in class. They were written with the intention of creating and maintaining a high level of cognitive demand throughout each task. The field notes recorded general observations of the entire class as well as the specific group chosen for close scrutiny during the study.

Prior to each lab, three external reviewers who had experience analyzing mathematical tasks using the Mathematical Tasks Framework (Stein, Smith, Henningsen, & Silver, 2000) were asked to determine the level of cognitive demand at which each of the labs was written. They evaluated the labs using a Mathematical Lesson Analysis Tool created by Day (2003). Once the Mathematical Lesson Analysis Tool was completed by each of the reviewers, the results were collected and the numerical scores for each level were averaged. Each lesson was characterized at the level of cognitive demand which had the highest average score associated with it. The level of cognitive demand determined by the

reviewers was important because so that it could be determined if the level of cognitive demand at which the students were engaged increased to a higher level, was maintained, or declined during implementation.

Group discussions were recorded using audiotapes and a cassette recorder. Every group in the classroom were recorded so that it is not evident to the students that only one group is the focus for the study. As instructor, the researcher moved around the classroom to observe what the groups were doing. All of the audiotapes were transcribed and analyzed to determine emerging themes. This also provided some general information about how the entire class progressed throughout the semester to ensure the validity of the specific case. The classroom activities were videotaped during the assigned labs in order to record non-verbal communications and confirm written field observations.

The individual student interviews took place after each cooperative lab during the semester. Merriam (1998) suggests that interviewing should be open-ended and semi-structured, so the researcher intends to have a set of pertinent questions to guide the interview. However, the order and the exact wording of the questions were not predetermined. Interviews lasted approximately 15 minutes and took place in the instructor's office. It was a course requirement for every student in the class to meet with the instructor following each lab to discuss their work. The interviews were recorded via audiotape and later transcribed. Student responses during each interview helped guide the development of questions for subsequent interviews during the semester.

Analysis

As the data were collected, a process of open coding was used to place phrases or groups of sentences into categories. Merriam (1998) describes this development of categories as an intuitive process that is systematic. The process was informed by the study's purpose, the researcher's orientation, and the meanings explicitly referenced by the participants themselves. The categories emerged based on similar ideas or comments made by the students. Related data were organized by lab and by sources. Since this process resulted in a large number of categories, the data were analyzed repetitively. Analysis of the categories that existed allowed for consolidation of the major ideas that were similar in nature. The final result was eight categories that reflect the prevalent terms and concepts derived from the collected data as indicated in Table 1.

Table 1.
Coding Categories

Category	Description	Data Sources
Recollection	Previously acquired knowledge supports application of new ideas and concepts.	Interviews, Field Observations
Cooperation	Interactions among group members encourage interpretation of results and challenge thinking.	Interviews, Field Observations
Construction	Conceptual understanding develops through exploration of mathematical processes.	Interviews, Transcriptions
Frustration	Students experience anxiety and	Interviews,

	insecurity in approaches to problem solving.	Transcriptions, Field Observations
Organization	Student work and approaches to problem solving are managed in clear manner.	Completed Labs, Interviews
Calculation	Numeric and symbolic computations are expedited and checked with features of the software.	Completed Labs, Interviews, Transcriptions
Representation	Students use various representations to analyze mathematical relationships.	Completed Labs, Field Observations, Transcriptions
Communication	Written explanations and justifications of mathematical processes allow for sharing of information.	Completed Labs, Transcriptions

Axial coding schemes were used to explore the interrelationship of categories (Creswell, 1998). Guided by the questions and purposes of the study, conditions surrounding each of the categories were examined to provide cumulative knowledge of connections between and among categories. First, the eight coding categories were separated into related groups. The Mathematical Tasks Framework suggests that the categories of Frustration and Recollection are characteristic of activities supporting the highest levels of cognitive demand. The categories of Organization, Calculation, Representation, and Communication all refer to how students make use of MathCAD in completing a mathematical task. The ways in which the cognitive computer tool was used both supported and structured their thinking. This allowed the students to explore the calculus concepts and encouraged high-level thinking.

The category of Cooperation proved to be more significant than originally anticipated. The cooperative atmosphere provided a support structure for implementing the use of technology. When one member of the group was unsure how to implement the technology, somebody else was there to assist them. Working in groups also enabled the students to engage in valuable mathematical discourse. The students would work together to recall previously learned concepts and apply them in a new context. The category of construction appeared to encompass all of the other categories. The cooperative experiences, uneasiness associated with complex problems, and requirement to apply previous knowledge all impacted student levels of cognitive demand. The use of the cognitive computer tool in completing the mathematical tasks allowed students to spend time struggling with difficult concepts without focusing on routine skills and procedures. This affects the way students approach their problem solving. Engagement at high-levels of cognitive demand allowed the students to construct shared mathematical meaning. Figure 2 depicts relationships between the categories discovered during the coding analysis.

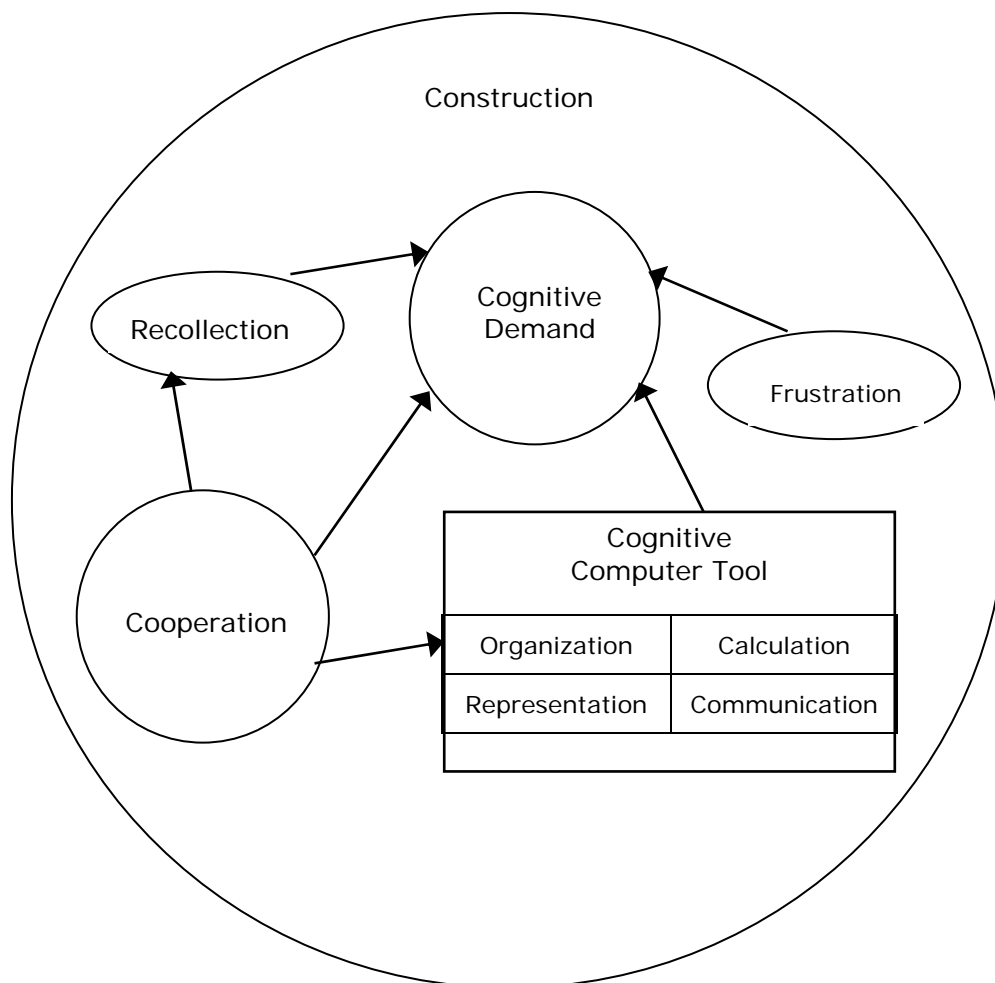


Figure 2. Relationships Between Categories
Conclusions and Implications

So what do the results of this study tell us with regards to the initial research questions? Let's begin with the secondary research questions: (1) What is the nature of students' use of the cognitive computer tools? (2) What are the students' perceptions about how the use of the cognitive computer tool contributes to their learning?

To answer Question 1, one can look at the four coding categories of Organization, Calculation, Representation, and Communication. The software enabled students to organize their work and justifications so that they could monitor their own progress in solving the problem. The capability of the software to perform numeric and symbolic computations was used throughout the labs by the group. The students focused less on number crunching and concentrated more on interpreting the results. MathCAD also allowed the group to create graphs, diagrams, equations, and written justifications together on the same worksheet. Students were able to analyze the different forms of information together and make connections. The software both promoted mathematical discussions and provided a means by which their work could be communicated electronically. Each of these

categories describes a way in which students implemented the feature of MathCAD in order to support their problem solving.

The answers to Question 2 came from the data collected during student interviews and exit questionnaires. In the interviews, the students reacted positively toward the use of the technology. The results from the exit questionnaire indicate that a majority of students believed the use of a computer algebra system allowed them to explore mathematics, spend more time focusing on interpreting results, and focus on understanding. The class responded overwhelmingly that the use of a computer algebra system is beneficial to learning.

The primary research question used to guide this study asked the following: In what ways does the use of a cognitive computer tool affect the level of cognitive demand on students in undergraduate calculus within the context of small group mathematical tasks? The categories of Frustration and Recollection are both referenced in the Mathematical Tasks Analysis Tool (Day, 2003) as indicators of high levels of cognitive demand. The high level of cognitive demand at which each of the labs was written was maintained during implementation with the exception of Lab 3. In Lab 3, there was a decline from the level of Procedures With Connections to a level of Procedures Without Connections. This did not appear to be an effect of using technology, but instead was caused by the group's inability to apply previous knowledge of high school geometry. They were left with little time to monitor their work and focused too much on the procedures for obtaining an answer. This resulted in logical mistakes and incorrect responses. In general, MathCAD provided support for developing conceptual fluency, aided in mathematical exploration, and allowed for the analyzing of different mathematical representations. The software tool took a focus off the arithmetic and tedious calculations so that they could spend more time discussing and trying to understand the inherent mathematical relationships and calculus topics, which were the focus of the course.

The results of this study have direct implications for mathematics educators at the undergraduate college level. This information can be valuable to those instructors who are or will be enhancing classroom experiences by integrating cognitive computer tools such as MathCAD. The study began with the hope that the results would indicate if the use of a cognitive tool had any effects on the level of student cognitive demand in which students engaged during mathematical tasks. It was discovered is that their focus was still primarily on solving problems and attempting to understand concepts. Most of the time, the use of the technology became an afterthought. The cooperative groups seemed to provide students with a support structure that enabled them to implement the technology effectively. While individual achievement was not the focus of this study, there were indications that the labs did help support understanding and ability on course assignments and tests.

At the end of the semester, an Exit Questionnaire was administered to the entire class. Twenty-six students completed the questionnaire. Students were asked to describe their perceptions of the role of MathCAD in the learning process. An overwhelming majority of the class agreed that the CAS allows them to explore mathematics, focus on interpreting results, and give more attention to the understanding of concepts. In addition, 89% of the class perceived the CAS to be beneficial to the learning of mathematics. A summary of each statement and the student responses is provides Table 2. The percentages represent the portion of the participants giving each possible response.

TABLE 2
Exit Questionnaire Results

	SA	A	N	D	SD
I am comfortable using computers or calculators in mathematical problem solving.	58%	42%	0%	0%	0%
I believe that using a computer algebra system in mathematics makes learning fun.	27%	27%	34%	4%	8%
I believe that the use of computer algebra systems in mathematics requires complex thinking.	12%	23%	15%	42%	8%
Using a computer algebra system in problem solving allows me to focus more on understanding mathematical concepts.	19%	46%	15%	19%	0%
A computer algebra system provides a means by which I can communicate mathematical ideas.	27%	42%	19%	12%	0%
Using a computer algebra system increases the ways that I can explore mathematics.	35%	57%	0%	8%	0%
Using a computer algebra system encourages me to memorize specific procedures to obtain a correct answer.	19%	35%	15%	27%	4%
Using a computer algebra system allows me to construct multiple representations of mathematical relationships.	27%	57%	12%	4%	0%
I believe that using a computer algebra system to perform routine calculations allows me to spend more time focusing on interpreting the results.	30%	50%	12%	8%	0%
I believe that using a computer algebra system is beneficial to learning mathematics.	35%	54%	4%	8%	0%

Key: SA-strongly agree, A-agree, N-no opinion, D-disagree, SD-strongly disagree

As the use of computers in the classroom increases, there is a need to move away from traditional teaching methods such as lecture and recitation. The use of MathCAD was critical to the teaching and learning philosophy in this undergraduate calculus class. It was an integral part of daily class work and this was reinforced in the cooperative labs given periodically during the semester. The cognitive computer tool reduced the amount of time spent on demonstrating low-level skills, allowed for creativity in problem solving, and permitted students to engage in high levels of cognitive demand. The coding categories discussed in this chapter can be used to identify technology-enhanced tasks that engage students at high-levels of cognitive demand. Jonassen (2000) coined the term "Mindtools" to describe a way of using a computer application program to engage students in constructive learning experiences. The tools themselves are not intelligent, but the learners definitely are. Jonassen (2005) argues that conceptual change takes place when technology is used for constructing models because of the intense cognitive and social activities involved.

As technology continues to work its way into increasingly more undergraduate classrooms, there are increased expectations concerning the incorporation of such technologies into mathematics teaching and learning. Computer tools such as MathCAD can become intellectual partners to support student learning. While this study supports the use of cognitive computer tools in a calculus classroom, it does not address the impact of these tools in undergraduate courses below calculus or in upper division mathematics courses. Mathematics colleagues are invited to continue further inquiry regarding appropriate uses of cognitive computer tools in undergraduate classroom settings. Future research could examine if the engaging of students in high levels of cognitive demand during technology-enhanced activities actually support long-term retention of mathematical understanding.

Appendix

MATH 1501 LAB #4 - Work on this assignment in your assigned groups. Show and justify all of your work using complete sentences when necessary. When you are done, one member of your group should send me your work via email as an attachment.

Type the name for each member of your group below:

- 1.
- 2.
- 3.
- 4.

- 1) In order to help ease traffic congestion in the city of Metropolis, the Metro Transportation Authority (MTA) wants to build a new monorail system that connects the two suburban communities of Jasper City and Village Park to a Downtown Station. The two communities are 17 miles apart on a line that is 14 miles due east of the downtown station as shown. Part of the proposed plan is to build an intermediate monorail station called Junction Station at which passengers can transfer to another monorail to take them to their respective communities. Your group has been asked to act as consultants for the MTA. Your job is to determine where Junction Station should be located relative to the Downtown Station so that the length of monorail track necessary to complete the project is minimized.

Steps we took:

1. we divided the distance from Jasper City to Village Park in half. Therefore, we get those two values to equal 8.5 miles
2. we drew a line from Junction Station to the midpoint of the distance from Jasper City to Village Park and made that value equal to x ; therefore, the distance from Downtown Station to Junction Station is $14-x$.
3. By using the 2 right triangles we find the distance from Junction Station to Village Park and Junction Station to Jasper City would be the value that we assigned to the variable h .

$$h = \sqrt{x^2 + 8.5^2}$$

$$f(x) := 14 - x + (2\sqrt{x^2 + 8.5^2})$$

Our first equation for the distance.

$$\frac{d}{dx}f(x) \rightarrow (-1) + \frac{2}{\left(x^2 + 72.25\right)^{\frac{1}{2}}} \cdot x$$

Then you must find the derivative of the equation and then set the derivative equal to zero and then solve for the critical numbers to search for a minimum value.

$$0 = -1 + \frac{2}{\left(x^2 + 72.25\right)^{\frac{1}{2}}} \cdot x \quad \text{has solution(s)} \quad 4.9074772881118189983$$

Now plug this answer into the equations $14 - x$ and the equation $\sqrt{x^2 + 8.5^2}$ and solve these equations for the answers.

So for the first equation $14 - x$ the answer is

$$14 - 4.9074772881118189983 \rightarrow 9.0925227118881810 \text{ (miles)}$$

Therefore the distance needed from downtown station to junction station is 9.0925227118881810017 miles.

- 2) Suppose a plant is 9mm tall at the beginning of an experiment. It is growing at a rate of $h(t) := t \cdot 2^{t-1}$ days after beginning the experiment. Find the value

of $\int_0^2 h(t) dt$ using your CAS and interpret the meaning of your answer.

$$\int_0^2 h(t) dt \rightarrow 4 \cdot \frac{2 \cdot \ln(2) - 1}{\ln(2)^2} + \frac{1}{\ln(2)^2}$$

$$4 \cdot \frac{2 \cdot \ln(2) - 1}{\ln(2)^2} + \frac{1}{\ln(2)^2} = 5.297$$

We plugged in the rate equation into the integral, on the interval of $[0,2]$. We solved for how many mm the plant grew over a period of 2 days. We found the answer to be 5.297 mm which is the distance the plant grew over 2 days. Therefore, the total distance the plant grew would be 9 mm + 5.297 mm = 14.297 mm

References

- Anand, P. G., & Zaimi A. (2000). Computer-managed instruction: Evaluation of alternate methods of technology integration in higher education. *Educational Technology & Society* 3(1), http://ifets.ieee.org/periodical/vol_1_2000/anand.html.
- Bauman, J. F., & Duffy, A. M. (2001). Teacher-researcher methodology: Themes, variations, and possibilities. *The Reading Teacher* 54(6), 608-615.
- Budenbender, J., Frischauf, A., Gogvadze, G., Melis, E., Libbrecht, P., & Ullrich, C. (2002). Using computers algebra systems as cognitive tools. In Proceedings of the 6th International Conference in Intelligent Tutoring Systems; Biarritz, France and San Sebastian, Spain, June 2-7, 2002. Berlin: Springer.
- Borchelt, N. A. (2004). *Effects of a computer tool on students' cognitive demand in undergraduate calculus*, Dissertation, Georgia State University (UMI No. 3150265).
- Confrey, J., Smith, E., Pilerio S., & Rizzutti, J. (1991). The use of contextual problems and multi-representational software to teach the concept of functions. (Final Rep. No. SE-052-920). Ithaca, NY: Cornell University, Department of Education.

- Crabill, C. D. (1990). Small-group learning in the Secondary mathematics classroom. In N. Davidson Ed.), *Cooperative learning in mathematics* (pp. 201-227). Menlo Park, CA: Addison-Wesley.
- Creswell, J. (1998). *Qualitative inquiry and research design*. Thousand Oaks, CA: Sage.
- Day, D. S. (2003). *Characterizing the level of cognitive demand of lessons in secondary mathematics textbooks*. Unpublished specialist's thesis. Georgia State University, Atlanta.
- Denzin, N. K., & Lincoln, Y. S. (1994). *Handbook of qualitative research*. Thousand Oaks, CA: Sage.
- Foletta, G. M. (1994). *Technology and guided inquiry: Understanding of students thinking while using a cognitive computer tool, the Geometer's Sketchpad, in a geometry class*. Dissertation, The University of Iowa (UMI No. 9433629).
- Ganguli, A. B. (1992). The effect on students' attitudes of the computer as a teaching aid. *Educational Studies in Mathematics*, 23, 611-618.
- Henningsen, M. A., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 29(5), 524-549.
- Humbree, R., & Dessart, D. J. (1992). Research on calculators in mathematics education. In NCTM (ed). *Calculators in mathematics education*. Reston, VA: Author.
- Johnson, D. W., Johnson, R. T. & Holubec, E. J. (1993). *Circles of learning: Cooperation in the classroom*. Edina, MN: Interaction Book Company.
- Jonassen, D. H. (2005). *Modeling with technology: Mindtools for conceptual change*. Third edition. New Jersey: Prentice Hall, Inc.
- Jonassen, D. H. (2000). *Computer as mindtools for schools: Engaging critical thinking*. Second edition. New Jersey: Prentice Hall, Inc.
- Jonassen, D. H., & Reeves, T. (1996). Learning with technology: Using computers as cognitive tools. In D. H. Jonassen (ed). *Handbook of Research on Educational Communications and Technology*. New York: Scholastic Press.
- MacLean, M. S. & Mohr, M. M. (1999). *Teacher-researchers at work*. Berkley, CA: National Writing Project, p. vii-ix.
- Merriam, S. (1998). *Qualitative research and case study applications in education*. San Francisco: Jossey-Bass, Inc.
- Noddings, N. (1990). Constructivism in mathematics education. In R.B. Davis, C.A. Maher & N. Noddings (Eds.) *Constructivist views on the teaching and learning of mathematics* (pp. 7-18), Reston, VA: NCTM.

- Pea, R.D. (1987). Cognitive technologies in mathematics education. In A. Schoenfeld (ed.), *Cognitive science and mathematics education* (pp. 89-122). Hillsdale, NJ: Lawrence Erlbaum Associates.
- Pitcher, N. (1998). Educational software in mathematics: Developing and using a mathwise module. *International Journal of Mathematics Education in Science and Technology* 29(5), 709-720.
- Ritchie, G. (2001). *The importance of teacher-research to the classroom teacher*. Retrieved March 19, 2003 from George Mason University, Graduate School of Education Web site: <http://www.gse.gmu.edu/research/tr/Trdefinition.shtml> .
- Rochowicz, Jr., J. A. (1996) The impact of using computers and calculators on calculus instruction: Various perceptions. *Journal of Computers in Mathematics and Science Teaching* 15(4), 423-435.
- Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School*, 3(4), 268-275.
- Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). *Implementing standards-based Mathematics instruction: A casebook for professional development*. New York: Teachers College Press.
- Tam, M. (2000). Constructivism, instructional design, and technology: Implications for transforming distance learning. *Educational Technology & Society* 3(2), http://ifets.ieee.org/periodical/vol_2_2000/tam.html.
- Vygotsky, L. S. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.
- Wenger, E. (1999). *Communities of Practice. Learning, meaning and identity*. Cambridge, MA: Cambridge University Press.
- Zbiek, (2003). Using research to influence teaching and learning with computer algebra systems. In J. Fey, Cuoco, C. Kieran, L. McMullin & R. M. Zbiek (Eds.), *Computer algebra systems in secondary school mathematics education* (pp. 197-216), Reston, VA: National Council of Teachers of Mathematics, Inc.